MATH 1A - MIDTERM 1

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Name: _____

Instructions: This midterm counts for 20% of your grade. You officially have 50 minutes to take this exam (although I will try to give you more time). This is a fairly long exam, so don't spend too much time on each question! Good luck, and don't worry, you'll be fine!

Note: Please check one (or more) of the following boxes if they apply to you:

- □ I am taking a Summer Session A course (May 23 - July 1), and I feel that this has prevented me from showing you my full math potential (this won't affect your midterm score, but I will keep this in mind when I'm assigning final grades)

- □ Today is the last day to add/drop this course. Please check this box if your decision of dropping this class depends on the score you'll receive on this exam, and you would like to have this exam graded by 5 pm (please be honest)

1		10
2		10
3		10
4		15
5		**40**
6		15
Bonus 1		5
Bonus 2		5
Total		100

Date: Friday, July 1st, 2011.

1. *(10 points)* Find the domain of $f(x) = \ln(x) + \sqrt{1 - x^2}$

2. *(10 points, 5 points each)* In the following problem, you do **not** have to graph the resulting functions. **BE BRIEF!**

   (a) Explain in words how to obtain the graph of $y = 2 - x^2$ from the graph of $y = x^2$

   (b) Explain in words how to obtain the graph of $y = \cos(2x + 3)$ from the graph of $y = \cos(x)$

Note: If you're stuck on how to write out your answer, here are some phrases you can use:
- Shift up/down/left/right by $\cdots$ units
- Stretch/Compress horizontally/vertically by a factor of $\cdots$
- 'Flip about the x/y -axis'.

3. *(10 points)* Find $f^{-1}(x)$, where $f(x) = 1 + e^{x^3}$

Note: Make sure to write your final answer in terms of x.

4. *(15 points)* Show $\tan(\cos^{-1}(x)) = \frac{\sqrt{1-x^2}}{x}$

Note: Show your steps. You are not just graded on the correct answer, but also on the way you write up your answer.

5. *(40 points, 5 points each)* Evaluate the following limits (or say 'it does not exist').

   Note: Show your work, except for (a) and (b), and do NOT use l'Hopital's rule

   (a) $\lim_{x\to\infty} \frac{1}{2x-1}$

   (b) $\lim_{x\to 3^+} \frac{e^x}{x-3}$

   (c) $\lim_{x\to 1} \frac{\sqrt{x+3}-2}{x-1}$

   (d) $\lim_{x\to\infty} \frac{x^2+2x+3}{x^2-2x+3}$

(e) $\lim_{x\to 2} \frac{x^2-4x+4}{x^2-3x+2}$

(f) $\lim_{x\to -\infty} \frac{\sqrt{x^2+1}}{x}$

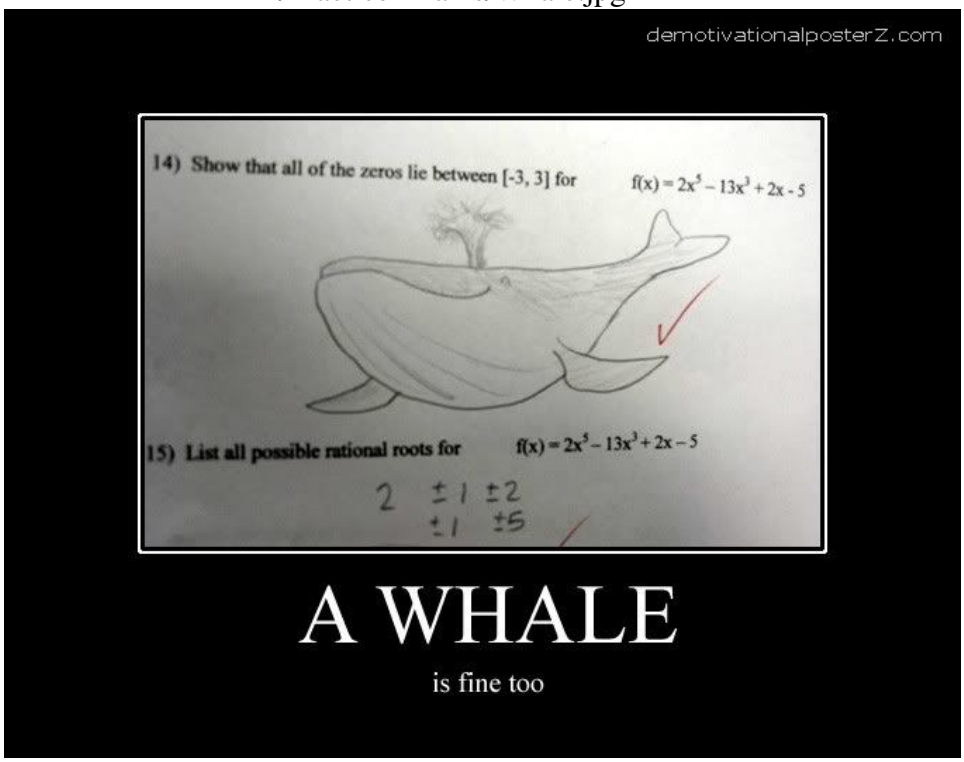
(g) $\lim_{x\to 0} \frac{|x|}{x}$

(h) $\lim_{x\to 0} x^4 \cos\left(\frac{4}{x}\right)$

6. *(15 points)* Show that the equation $x^3 + x - 1 = 0$ has at least one solution.

Note: Show your work! You will be graded not only on the correct answer, but also on the way you write up your answer

1A/Practice Exams/Whale.jpg



Bonus 1 *(5 points)* Suppose you are taking an elevator up from the 1^{st} floor to the 3^{rd} floor, and, at the same time, your friend is taking another elevator down from the 3^{rd} floor to the 1^{st} floor. Assume that the motion of the elevators is continuous, and that both of you get out of your elevators after 1 minute. Show that at some point in time you two are on the same level. (So theoretically, at that moment you can wave at your friend)

Hint: Let $g(t)$ be the height of your elevator at time t, and let $h(t)$ be the height of your friend's elevator at time t. Consider the function $f(t) = g(t) - h(t)$

Bonus 2 *(5 points)* Let f be any function.

(a) Show that the function $g(x) = \frac{f(x)+f(-x)}{2}$ is always even.

(b) Show that the function $h(x) = \frac{f(x)-f(-x)}{2}$ is always odd.

(c) Using (a) and (b), show that any function f can be written as a sum of an even function and an odd function. (this is called the **even/odd decomposition of** f, and in fact it is unique)

(d) If $f(x) = e^x$, calculate g and h as in (a). Do you happen to know the names of those two functions?

Isn't this problem really cool? :)

Any comments about this exam? (too long? too hard?)